Calculation of Internal Flows Using a Single-Pass, Parabolized Navier-Stokes Analysis

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A single-pass, subsonic parabolized Navier-Stokes (PNS) procedure is developed to analyze three-dimensional viscous internal flows. Elliptic effects are incorporated by initializing the PNS code with the streamwise pressure gradient distribution obtained from a coarse grid Euler calculation. When the Euler code incorporates the transport of a "viscous-like" inlet vorticity profile, the rotational inviscid pressure distribution provides a good initialization for the single-pass PNS procedure. Computed primary flow velocities for laminar flow in a curved duct of square cross section agree well with experimental data.

Introduction

THIS article, based on Ref. 1, describes the key features of a single-pass parabolized Navier-Stokes (PNS) procedure, developed by the authors, for efficiently calculating internal flows with secondary motion. The code incorporates a finite-volume pressure correction formulation due to Rhie. Numerical details appear in Refs. 1 and 2.

Many researchers have developed PNS solution strategies for computation of internal flows of varying complexity. A brief overview of these is included in Ref. 1. PNS approaches have in common that streamwise diffusion terms are neglected in the full Navier-Stokes equations, and streamwise pressure gradient terms are decoupled from the cross-stream pressure gradients. These approximations give rise to computationally efficient space marching solution strategies. In the present approach, the transverse distribution of a streamwise pressure gradient is obtained by interpolation from a coarse grid Euler computation, the viscous effects then being resolved by a single-pass of the PNS solver through a fine-grid computational domain.

The development of secondary flow in curved ducts has been studied extensively.[4-6] In incompressible flow, such secondary motions are generated by the rotational inviscid deflection of transverse vorticity present at the inlet and also by the deflection of transverse vorticity generated within the wall boundary-layer region. The deflection of these vorticities can generate strong transverse pressure gradients and influence the streamwise pressure gradient. When this influence becomes too strong, the single-pass PNS calculation becomes inadequate, because of pressure decoupling. In practical engineering applications, the streamwise pressure gradient modification due to inlet vorticity deflection is far more important than the streamwise pressure gradient modification due to additional vorticity deflection arising from boundary-layer growth. In the present single-pass PNS calculations, therefore, the former influence is incorporated in the Euler calculation used to initialize the pressure field.

This approach was applied to the calculation of incompressible laminar flow through a square duct, with a 90 deg bend to verify its ability to predict strong secondary flows in the absence of turbulence modeling errors. This $R_{eD} = 7.9 \times 10^2$ flow configuration was tested experimentally by Humphrey et al.[3] The duct has a mean radius of curvature to hydraulic diameter ratio of 2.3.

The three-dimensional elliptic Euler solver used for initialization is a member of the same family of pressure-correction-method based flow solvers as the PNS code used herein, and a full Navier-Stokes (FNS) solver. Incompressible Euler solutions were obtained on a $9 \times 17 \times 30$ computational grid, both with and without specification of an analytically obtained fully developed inlet vorticity profile. The inviscid transport of inlet cross-stream vorticity was seen to have a significant effect on the pressure distribution in the inviscid flowfield (see Ref. 1 for more detail). When a flat inlet velocity profile was specified, the Euler pressure field exhibited two-dimensional potential character, as expected.

Two incompressible PNS solutions were obtained on a $21 \times 31 \times 88$ computational mesh, shown in Fig. 1. Fully developed inlet velocity profiles were specified for each run, however the two different Euler pressure fields were imposed. Specifically, the cross-stream distribution of streamwise pressure gradient was interpolated onto the fine PNS grid. A global one-dimensional correction to the inviscid level of streamwise pressure gradient is applied to preserve inlet mass flow rate, and these corrected values are used in the discretized PNS equations. In Fig. 2, computed primary velocity magnitudes at the bend exit are compared with the experimental data of Ref. 3. For this configuration, the single-pass PNS method predicts the primary flow with reasonable accuracy, if an inviscid pressure field which incorporates inlet vorticity transport is used to initialize the calculation. The potential flow-like pressure initialization is seen to be inadequate for initializing a single-pass PNS solution. Figure 3 shows predicted secondary flow vectors at the bend exit for the two differently initialized PNS runs. Stronger and more complex secondary motions are predicted when a rotational initialization is used.

The single-pass PNS method captured the primary flow distribution for the present case, but is less computationally intensive than multiple-pass PNS and FNS approaches. Other researchers have applied multiple-pass PNS[2] and FNS[6] solution procedures to compute the present case. In both cases, primary flow velocities are well predicted. A single pass of the PNS solver requires approximately an order of magnitude less CPU time to converge than the FNS solver does for the same case. Convergence is measured by a two order-of-magnitude drop in total local mass balance residual. The computational mesh for the background Euler calculation is generally less refined than the viscous grid for a particular calculation, and

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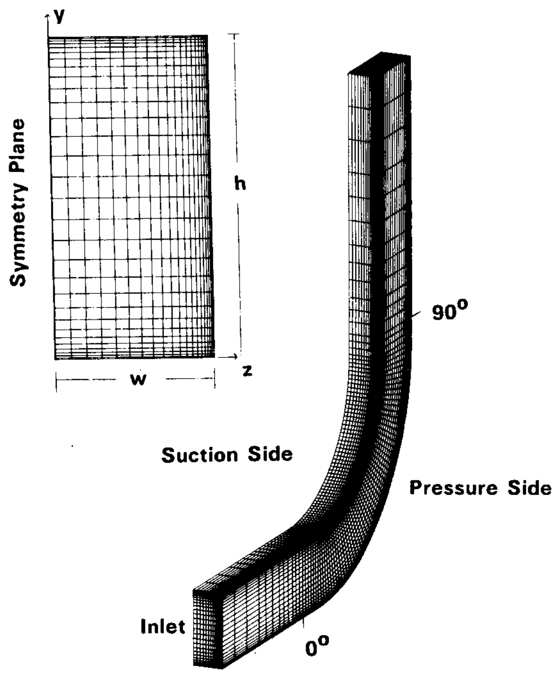

Fig. 1 PNS computational mesh; 21×31×88.

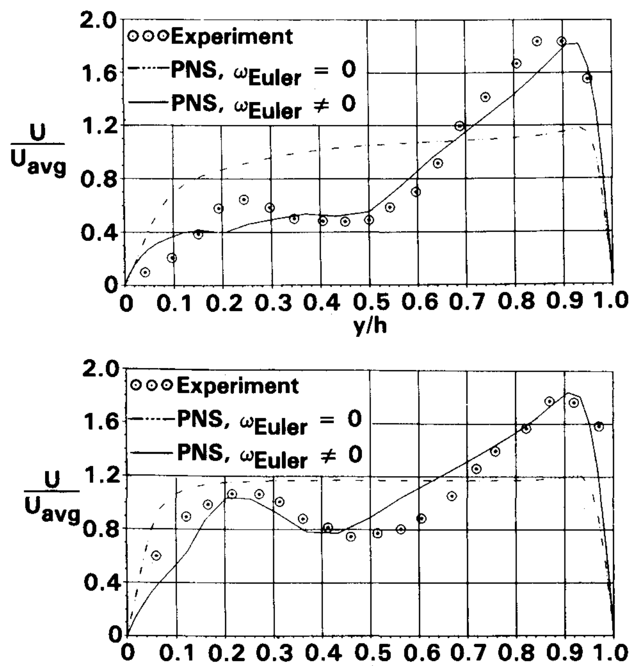

Fig. 2 Primary velocity at 90 deg station. Calculation vs experiment at $z/w = 0.0$ (top), 0.5 (bottom).

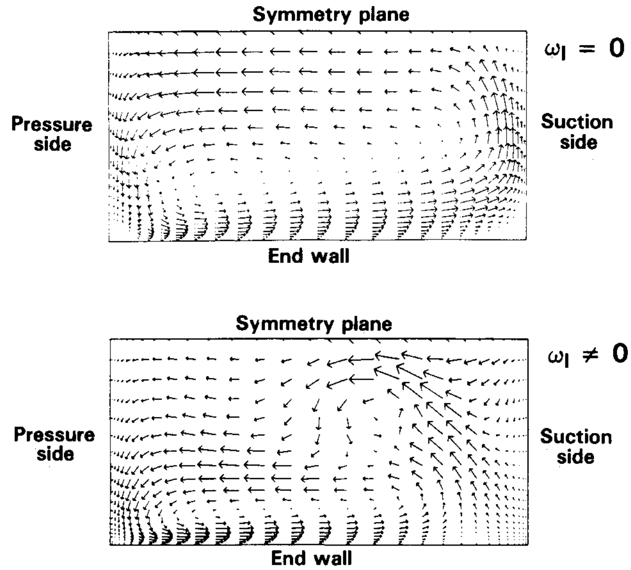

Fig. 3 Secondary flow vectors at 90 deg station. Irrotational (top) and rotational (bottom) Euler initialization.

viscous terms are not incorporated. The initialization, then, accounts for only about as much CPU time as the PNS sweep for a net savings of the present analysis procedure over a full Navier-Stokes procedure of approximately an order of magnitude.

The net CPU savings of the present approach is accompanied by several limitations. The PNS code is not applicable in regions of large flow reversal. In the present case, experiment and computations (Refs. 1-3 and 7) predict the presence of a small pocket of flow reversal in the outer-wall pressure-surface corner between $O = 0$ deg and 45 deg along the bend. The present PNS code can march through this relatively small region by implementation of the FLARE approximation; however, in highly curved ducts at low Reynolds numbers, large regions of flow reversal may be present and the PNS approach rendered inapplicable. Even though the present PNS code showed good analytical capability for a curved duct with strong transverse vorticity, the success came from the use of engineering judgment in setting up the computational domain and the Euler initial pressure. If the streamwise pressure gradient modification due to the interaction between boundary-layer growth and the passage curvature becomes too strong, a single-pass PNS calculation becomes inadequate.

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